Lesson 27
Solve Problems with Cylinders, Cones, and Spheres

Prerequisite: Find the Volume of Cones and Cylinders

Study the example problem showing how to find the volume of a cylinder and a cone. Then solve problems 1–7.

Example
Find the volumes of the cone and the cylinder. Write the volumes in terms of $\pi$.

<table>
<thead>
<tr>
<th>Volume of Cone</th>
<th>Volume of Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
<td>$V = \pi r^2 h$</td>
</tr>
<tr>
<td>$= \frac{1}{3} \pi \cdot 6^2 \cdot 5$</td>
<td>$= \pi \cdot 6^2 \cdot 5$</td>
</tr>
<tr>
<td>$= 60\pi$</td>
<td>$= 180\pi$</td>
</tr>
</tbody>
</table>

The volume of the cone is $60\pi$ cubic centimeters, and the volume of the cylinder is $180\pi$ cubic centimeters.

1. In the example, what measurements are the same in the cone and the cylinder?

They both have a radius of 6 cm and a height of 5 cm.

2. How does the volume of the cone in the example compare to the volume of the cylinder?

The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder.

3. Suppose the height of the cone in the example was tripled. How would the volume of the cone compare to the volume of the cylinder? Explain how you know.

The volumes would be equal. Possible explanation: The volume of the cone is $\frac{1}{3}$ of the volume of the cylinder. If the height of the cone were tripled, the volume would be tripled, and $3 \cdot 60\pi = 180\pi$.

Vocabulary
- **cylinder**: a solid figure with two congruent and parallel circular bases.
- **cone**: a solid figure with one vertex and one circular base.

Solve.

Use the figures for problems 4–6.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{h}$</td>
<td>$\text{2h}$</td>
<td>$\text{2h}$</td>
<td>$\text{2h}$</td>
</tr>
</tbody>
</table>

4. Find the volume of each solid figure. Write your answers in terms of $\pi$, $r$, and $h$.

A: $\frac{1}{3} \pi r^2 h$  
B: $\frac{8}{3} \pi r^2 h$  
C: $8\pi r^2 h$  
D: $\frac{2}{3} \pi r^2 h$

5. Order the solid figures from least volume to greatest volume. Explain your reasoning.

Solid D, solid A, solid B, solid C; Possible explanation: All of the volumes have the term $\pi r^2 h$, so I compared the coefficients: $\frac{2}{3} < 1 < \frac{8}{3} < 8$.

6. Sheila says that a cone with a radius of $2r$ and a height of $6h$ would have the same volume as one of the solid figures above. Do you agree with Sheila? Explain why or why not.

Yes, I agree with Sheila. Possible explanation: It would have the same volume as solid C because $\frac{1}{3} \pi (2r)^2 \cdot 6h = \frac{24}{3} \pi r^2 h = 8\pi r^2 h$.

7. A cylinder and a cone have the same radius. The volume of the cone is twice the volume of the cylinder. How many times greater is the height of the cone than the height of the cylinder? Explain your reasoning.

The height of the cone is 6 times greater than the height of the cylinder. Possible explanation: The volume of a cone with the same radius and height as a cylinder is $\frac{1}{3}$ of the volume of the cylinder. A cone with the same volume as the cylinder has a height that is 3 times the height of the cylinder. A cone with twice the volume has a height that is $2 \cdot 3$, or 6, times the height of the cylinder.
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Solve Volume Problems

Study the example problem showing how to solve a volume problem. Then solve problems 1–8.

Example

Two solid glass paperweights are shown at the right. What is the volume of glass used to make each paperweight? Write the volumes in terms of π.

### Volume of Cone

\[ V = \frac{1}{3} \pi r^2 h \]

### Volume of Cylinder

\[ V = \pi r^2 h \]

The volume of glass used in the cone paperweight is \( \frac{1}{3} \pi r^2 h \) cubic centimeters, and the volume of glass used in the cylinder paperweight is \( \pi r^2 h \) cubic centimeters.

In the example, was more glass used to make the cone paperweight or the cylinder paperweight? How much more?

- **the cylinder paperweight:** \( 16\pi \) cubic centimeters

In the example, why was 4 and not 8 substituted for \( r \) to find the volume of the cylinder?

The volume formula uses the radius, not the diameter. The diameter of the cylinder is 8 cm; the radius is 4 cm.

The paperweight manufacturer decides to change the height of the cone paperweight in the example so that the same volume of glass is used to make both paperweights. What will be the new height of the cone paperweight? Explain your reasoning.

- **9 cm:** Possible explanation: The paperweights have the same radius, so a cone with a height that is 3 times the height of the cylinder will have the same volume. \( 3 \times 3 \text{ cm} = 9 \text{ cm} \).

Solve.

M 4

Find the volume of the cylinder-shaped grain storage tank at the right. Write the volume in terms of \( \pi \).

\[ 200\pi \text{ cubic meters} \]

M 5

Find the volume of the cone-shaped grain storage tank at the right. Write the volume in terms of \( \pi \).

\[ 100\pi \text{ cubic meters} \]

M 6

For the tanks in problems 4 and 5, calculate each volume using 3.14 for \( \pi \).

- **Cylinder-shaped tank:** 628 cubic meters
- **Cone-shaped tank:** 314 cubic meters

M 7

Jacob is buying cups for his frozen yogurt business. He can choose from the cups shown at the right. He wants to buy the type that will hold more frozen yogurt.

a. Find the volume for each type of cup. Use 3.14 for \( \pi \), and round to the nearest tenth.

- **cylinder:** 28.3 cubic inches
- **cone:** 25.1 cubic inches

b. Which type of cup should Jacob buy? Explain.

- **He should buy the cylinder-shaped cups because they will hold more frozen yogurt.**

C 8

The height of a cylinder is 6 centimeters and the volume is 150\( \pi \) cubic centimeters. A cone has the same volume as the cylinder. Give two possible radius and height combinations for the cone: one with the same radius as the cylinder, and one with a different radius. Explain your reasoning.

- **Possible answers:** radius 5 cm and height 18 cm, radius 15 cm and height 2 cm; Possible explanation: The radius of the cylinder is 5 cm, so I used 5 cm as the radius of the cone and calculated that 18 cm would be the height with the same volume. Then I tripled 5 cm to use 15 cm for the radius and calculated that 2 cm would be the height.
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| Name: __________________________ |

Compare Volumes

Study the example showing how to solve problems involving volume. Then solve problems 1–6.

Example

An artist is making three solid figures out of clay. The designs for the solid figures are shown at the right. Find the volume of all three figures.

Use $\pi = 3.14$, and round the answers to the nearest hundredth. Which figure requires the most clay?

<table>
<thead>
<tr>
<th>Sphere</th>
<th>Cylinder</th>
<th>Cone</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V = \frac{4}{3} \pi r^3$</td>
<td>$V = \pi r h$</td>
<td>$V = \frac{1}{3} \pi r^2 h$</td>
</tr>
<tr>
<td>$= \frac{4}{3} \pi (2)^3$</td>
<td>$= \pi r h$</td>
<td>$= \frac{1}{3} \pi r^2 h$</td>
</tr>
<tr>
<td>$= \frac{4}{3} (3.14)(2)^3$</td>
<td>$= 3.14 \cdot (1.5)^2 \cdot 4$</td>
<td>$= \frac{1}{3} (3.14)(3)^2(3)$</td>
</tr>
<tr>
<td>$= 33.49 \text{ in.}^3$</td>
<td>$= 28.26 \text{ in.}^3$</td>
<td>$= 28.26 \text{ in.}^3$</td>
</tr>
</tbody>
</table>

The volume of the sphere is the greatest and therefore requires the most clay.

How much clay does the artist need to make all three figures?

90.01 in.$^3$

If the artist begins with a spherical ball of clay with a radius of 3 inches, does she have enough clay to make all three figures? Explain.

Yes; Possible explanation: The volume of clay at the beginning is $\frac{4}{3} \pi (3)^3 = 113.04 \text{ in.}^3$, which is greater than the sum of the volumes of the three figures.

Does the artist have enough clay to make three of the spheres shown instead of one of each shape? Explain.

Yes; Possible explanation: $113.04 \div 33.49 \approx 3.38$, so she would have enough clay to make three of the spheres shown.

Solve.

M 4 Two bowls are half-spheres. Find the volumes of the bowls. Use 3.14 for $\pi$, and round the answers to the nearest whole number. How many times greater is the volume of the large bowl than the volume of the small bowl?

Small bowl |
| Large bowl |
| $V = \frac{1}{2} \pi r^2 h$ | $V = \frac{1}{2} \pi r^2 h$ |
| $= \frac{1}{2} \pi \cdot 3.14 \cdot 1^2$ | $= \frac{1}{2} \pi \cdot 3.14 \cdot 2^2$ |
| $= \frac{1}{2} \cdot 3.14 \cdot 4$ | $= \frac{1}{2} \cdot 3.14 \cdot 8$ |
| $= \frac{1}{2} \cdot 3.14 \cdot 64$ | $= \frac{1}{2} \cdot 3.14 \cdot 512$ |
| $= 134$ | $= 1,072$ |

Solution: The volume of the large bowl is 8 times greater than the volume of the small bowl.

Use the given information and the figures shown to solve problems 5–6.

A toy company makes sphere-shaped and cone-shaped toys. Two of the toys are shown at the right.

M 5 Lawrence says that because the height and radius of the cone are equal to the radius of the sphere, the two toys have the same volume. Explain why Lawrence is incorrect.

Possible answer: The volume of the sphere is $\frac{4}{3} \pi (9)^3 = 3,052 \text{ cm}^3$.

The volume of the cone is $\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (9)^2 (9) = \frac{1}{3} \pi (9)^3 = 763 \text{ cm}^3$.

M 6 Change one of the dimensions of the cone-shaped toy so that the volumes of the toys are the same.

Possible answer: Change the height of the cone-shaped toy to 36 cm.

The volume is then $\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (9)^2 (36) = 3,052 \text{ cm}^3$. 

C 8 Change one of the dimensions of the cone-shaped toy so that the volumes of the toys are the same.

Possible answer: Change the height of the cone-shaped toy to 36 cm.

The volume is then $\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (9)^2 (36) = 3,052 \text{ cm}^3$. 

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Solve Problems with Cylinders, Cones, and Spheres

Solve the problems.

1. The propane storage tank shown is a cylinder with a half-sphere on each end. Tell whether each statement is True or False.
   a. The volume of the cylinder part of the tank is about 550 ft³. True False
   b. The volume of one of the half-spheres is about 65 ft³. True False
   c. The combined volume of the two half-sphere parts is less than the volume of the cylinder part. True False
   d. The volume of the tank is about 202 ft³. True False

2. Tom is making strawberry jelly and is going to put it into the jar shown. About how much jelly will he need to fill the jar to 0.5 inch from the top? Circle the correct answer. (Use 3.14 for π, and round to the nearest whole)
   A 28 in³
   B 57 in³
   C 63 in³
   D 127 in³

   Isabelle chose C as the correct answer. How did she get that answer? Possible explanation: Isabelle used 5 inches for the height instead of 4.5 inches.

3. Juan cut a section shaped like a cone out of the center of a piece of wood that is shaped like a cylinder, as shown below. What is the volume of the piece of wood left after Juan cut out the cone-shaped section? Use 3.14 for π, and round to the nearest whole cubic centimeter.
   Show your work.
   Cylinder
   \[ V = \pi r^2 h \]
   \[ = 3.14 \cdot 5^2 \cdot 12 \]
   \[ = 3.14 \cdot 25 \cdot 12 \]
   \[ = 3.14 \cdot 300 \]
   \[ = 3014 \approx 3015 \]
   Cone
   \[ V = \frac{1}{3} \pi r^2 h \]
   \[ = \frac{1}{3} \cdot 3.14 \cdot 5^2 \cdot 12 \]
   \[ = \frac{1}{3} \cdot 3.14 \cdot 25 \cdot 12 \]
   \[ = \frac{1}{3} \cdot 376.8 \]
   \[ = \frac{376.8}{3} \approx 125.6 \]
   Solution: The volume of the piece of wood left is about 2,925 cm³.

4. A barrel in the shape of a cylinder is cut in half lengthwise to make a water trough for horses. Does the expression \( \frac{1}{2} \pi r^2 h \) represent the volume of water that the water trough holds? Explain why or why not.
   No; Possible explanation: That expression gives the volume of a complete cylinder with half the radius. The water trough is half of the cylinder, so the formula is just half the volume of a cylinder, or \( \frac{1}{2} \pi r^2 h \).

5. The barrel in problem 4 is 5 feet long and the radius of its base is 1.5 feet. How much water will the water trough hold? Use 3.14 for π, and round to the nearest whole cubic foot.
   \[ V = \frac{1}{2} \pi r^2 h = \frac{1}{2} \cdot 3.14 \cdot 1.5^2 \cdot 5 \]
   \[ = 17.7 \] The water trough will hold about 18 ft³ of water.